Universal behavior of one-dimensional multispecies branching and annihilating random walks with exclusion

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(Received 30 October 2000; published 16 April 2001)

A directed percolation process with two symmetric particle species exhibiting exclusion in one dimension is investigated numerically. It is shown that if the species are coupled by branching ($A \to AB$, $B \to BA$), a continuous phase transition will appear at the zero-branching-rate limit belonging to the same universality class as that of the two component branching and annihilating random-walk model with two symmetric offsprings. This class persists even if the branching is biased towards one of the species. If the two systems are not coupled by branching but a hard-core interaction is allowed only the transition will occur at finite branching rate belonging to the usual $(1+1)$-dimensional directed percolation class.

DOI: 10.1103/PhysRevE.63.056108 PACS number(s): 05.70.Ln, 82.20.Wt, 64.60.−i, 05.40.−a

The study of phase transitions in low dimensions is an interesting and widely investigated topic [1,2]. Research on nonequilibrium phase transitions occurring in one-dimensional coupled systems has nowadays drawn interest [3–14]. Several models have been found with transitions that do not belong to the robust directed percolation (DP) class [15–17] or to the parity conserving (PC) class [18,19] which are the most prominent ones among one-component systems. Particle blocking which is common in one dimension has not been taken into account in field theoretical descriptions of these models yet [20,21]. It has been known for some time that the pair contact process [24] can be regarded as a coupled system that exhibits DP class static exponents while the spreading ones depend on initial densities [25]. The field theoretical investigation of Janssen [21] predicts that in coupled DP systems the symmetry between species is unstable and generally a phase transition belongs to the class of unidirectionally coupled DP where coupling between pairs of species is relevant in one direction only. Such systems have been shown to describe also certain surface roughening processes [9,10].

Recently we have shown [22] that in the two-component annihilating random walk ($AA \to \emptyset$, $BB \to \emptyset$) owing to the hard-core interaction of particles dynamical exponents are nonuniversal. Some consequences of hard-core effects for random walks in one dimension have been known for some time already [23].

Very recently simulations [6,7] gave numerical evidence that in the two-component branching and annihilating random walk (2-BARW2) the lack of particle exchange between different species results in new universality classes in contrast to widespread beliefs that bosonic field theory can well describe these systems. The critical exponents obtained numerically suggest that the location of offspring particles at branching is the relevant factor that determines the critical behavior. In particular if the parent separates the offsprings (i) $A \to BAB$, the steady-state density will be higher than in the case when they are created on the same site, and (ii) $A \to ABB$, for a given branching rate because in the former case they are unable to annihilate with each other. This results in different order parameter exponents for the symmetric (2-BARW2s) and the asymmetric (2-BARW2a) cases [$\beta_s = 1/2$ vs $\beta_a = 2$ for (i) and (ii), respectively].

Hard-core effects are conjectured to cause a series of new universality classes in one dimension [6]. In this paper I point out that probably only a few universality classes emerge as the consequence of particle exclusion, because other symmetries and conservation laws (like that of the PC class) will become irrelevant.

In the present study first I show that in case of the two-component single off-spring BARW model (2-BARW1), defined as

$$A \xrightarrow{\sigma_A/2} AB, \quad A \xrightarrow{\sigma_A/2} BA, \tag{1}$$

$$B \xrightarrow{\sigma_B/2} BA, \quad B \xrightarrow{\sigma_B/2} AB, \tag{2}$$

$$AA \xrightarrow{\lambda} \emptyset, \quad BB \xrightarrow{\lambda} \emptyset, \tag{3}$$

$$A\emptyset \overset{d}{\leftrightarrow} \emptyset A, \quad B\emptyset \overset{d}{\leftrightarrow} \emptyset B, \tag{4}$$

$$AB \overset{0}{\leftrightarrow} BA, \tag{5}$$

a continuous phase transition will occur at zero branching rate limit ($\sigma = 0$) like in the 2-BARW2 model where they are equivalent and therefore the exponents on the critical point must be the same as those determined in [7,22]. Furthermore, I show that the order parameter exponent describing the singular behavior of the steady-state density near the critical point coincides with that of the 2-BARW2s model.

The particle system was simulated on a lattice with size $L = 4 \times 10^4$ and periodic boundary conditions for different σ's (with $\lambda = d = 1 - \sigma$ condition). The initial condition was a uniformly random distribution of A's and B's with a total concentration of 0.5. The evolution of the density was followed until steady state has been reached plus $t \sim 10^4$ Monte Carlo sweeps [throughout the whole paper t is measured in units of Monte Carlo sweeps (MCS) of the lattices]. As Fig. 1 shows a phase transition occurs at $\sigma_A = \sigma_B = 0$ indeed.

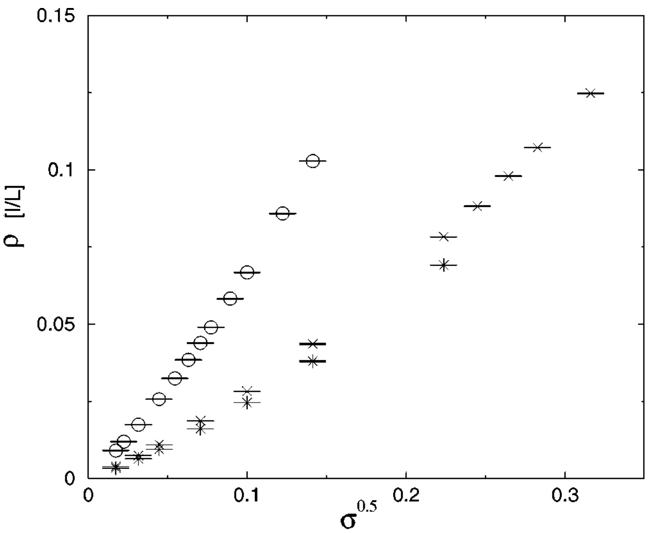

FIG. 1. Steady-state density as a function of $\sigma^{0.5}$ in the one-dimensional 2-BARW1 model. Circles correspond to $\rho_A+\rho_B$ for $\sigma_A=\sigma_B$, crosses to ρ_A, and stars to ρ_B when $\sigma_A=\sigma_B/2$.

The order parameter exponent has been determined with a local slope analysis of the data,

$$\beta_{eff}(\sigma)=\frac{\ln \rho_i-\ln \rho_{i-1}}{\ln \sigma_i-\ln \sigma_{i-1}}, \tag{6}$$

providing an estimate for the true asymptotic behavior of the order parameter:

$$\beta=\lim_{\sigma\to 0}\beta_{eff}(\sigma). \tag{7}$$

As one can see in Fig. 2 β_{eff} extrapolates to $\beta=0.50(1)$ with a strong correction to scaling like in case of the 2-BARW2s model [7]. The coincidence of this off-critical exponent in addition to the equivalence of processes at the critical point assures that they belong to the same universality class.

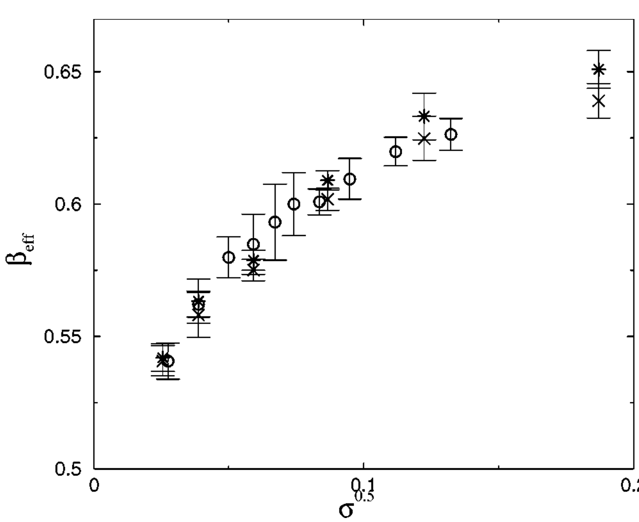

FIG. 2. Effective β in the 2-BARW1 model as a function of $\sigma^{0.5}$. Different symbols denote the same as in Fig. 1.

If we destroy the symmetry between species by the branching rates $\sigma_A=\sigma_B/2$, we still get the same order parameter exponents $[\beta=0.50(1)]$ for both species (Fig. 2). Therefore this universality class is stable with respect to coupling strengths unlike colored and flavored directed percolation [21].

It is also insensitive to whether or not the parity of particles is conserved, meaning that the $A\to BAB$ process can be decomposed into a sequence of $A\to AB$, $AB\to BAB$ processes. This may seem to be quite obvious when particle exchange is not allowed and if locality is assumed. By the choice of parameters $d=1-\sigma$ in the neighborhood of the critical point the diffusion is strong and the locality condition is not met. Still the two processes share the same critical behavior.

If we decouple the two systems and allow hard-core exclusion only,

$$A\stackrel{\sigma}{\to}AA, \tag{8}$$

$$B\stackrel{\sigma}{\to}BB, \tag{9}$$

$$AA\stackrel{\lambda}{\to}\emptyset, \quad BB\stackrel{\lambda}{\to}\emptyset, \tag{10}$$

$$A\emptyset\stackrel{d}{\leftrightarrow}\emptyset A, \quad B\emptyset\stackrel{d}{\leftrightarrow}\emptyset B, \tag{11}$$

$$AB\stackrel{0}{\leftrightarrow}BA, \tag{12}$$

the critical point will be shifted to $\sigma=0.811\,07(1)$ and DP-like density decay can be observed on the local slopes defined as

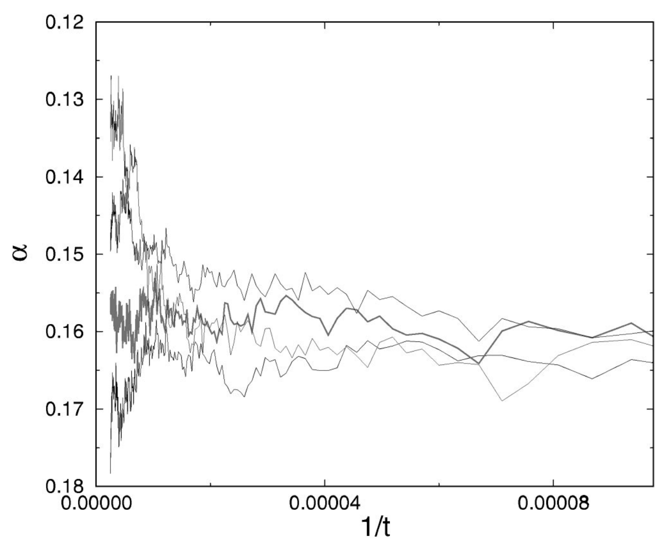

FIG. 3. Effective decay exponent α_{eff} as the function of $1/t$ in the decoupled two-component DP model. The system size is $L=4\times 10^4$; the decay is followed for 2×10^5 MCS. The different curves correspond to $\sigma=0.81103$, 0.81107, 0.81109, 0.8111 (from bottom to top).

$$\alpha_{eff}(t) = \frac{-\ln[\rho(t)/\rho(t/m)]}{\ln(m)} \tag{13}$$

(where we use $m=8$ usually) (see Fig. 3).

One cannot observe any relevant correction to scaling here; the most straight curve corresponding to the critical one ($\sigma=0.81107$) extrapolates to $\alpha=0.158(2)$ which agrees very well with the $\beta/\nu_{||}=0.159\,464(6)$ value of the $1+1$ DP class value that can be found in the literature [26]. This is different from the case of two species annihilating random walk with exclusion, where the particle blocking causes marginal perturbation to the standard decay process [22].

One can generalize the results by taking into account that neighboring AA and BB offsprings decay very quickly and are therefore irrelevant for the leading scaling behavior.

Conjecture: In coupled, one-dimensional N-component BARW systems with particle exclusion and branching processes like A→BABB, A→BAAA, A→BAC, . . . , leaving behind nonreacting neighboring particles which block each other, the universality class of a phase transition will be the same as that of 1-BARW2s. If the branching creates only pairs that can annihilate immediately (like A→BAAB, . . .), the class of transition will be the same as that of the 2-BARW2a model. We can also conclude that in the case of reaction-diffusion processes where spontaneous decay is allowed, $2A\to A$, $A\to\emptyset$, the blocking effect between dissimilar species is irrelevant.

It is very likely that the transition of a very recently introduced ladder model [8] also belongs to this class. This model is composed of two one-dimensional subsystems following BARW at the critical point and coupled by ladder links. In the supercritical region, by updating an active site one can create an offspring on the other subsystem or increase the inactivity level of that site. For small coupling strength ($s=1$) the very few blocking events cannot introduce relevant blocking on the other subsystem and the scaling exponents agree with those of the coupled BARWe model without exclusion [20]. For stronger coupling strength ($s=2$) there are more blocking possibilities resulting in 1-BARW2s scaling exponents.

In conclusion I have shown that the one-dimensional two-species coupled BARW with exclusion and one offspring has the same critical transition point as that of the 2-BARW2s model investigated earlier. The hard-core interaction itself is not sufficient to cause a deviation in the scaling behavior from that of DP. A conjecture is given with regard to the universality classes in coupled BARW systems exhibiting particle exclusion.

Note added in proof. Recently work by Kwon and Park (e-print cond-mat/0010381) investigating numerically the 2-BARW1 model appeared.

The author would like to thank T. Antal for the stimulating discussions and N. Menyhárd for critically reading the manuscript. Support from Hungarian research fund OTKA (Nos. T-25286 and T-23552) and from Bolyai (No. BO/00142/99) is acknowledged.

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